Combining Discrete Morse Theory with more traditional differential tools to study the geometry and topology of gravity in sampled space

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Background

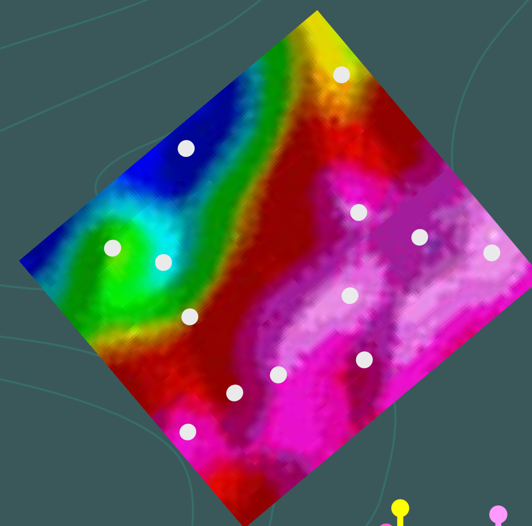
- Differential Geodesy (DG)
- Forward problem: from a hypothetical mass distribution to a computed gravitational field
- Inverse problem: from gravity measurements to a compatible mass distribution
- DG in prospecting: inverse problem focused on detection of signature density distributions
- DG in geodynamics: relating gravity changes to expected (or unsuspected) mass re-distribution

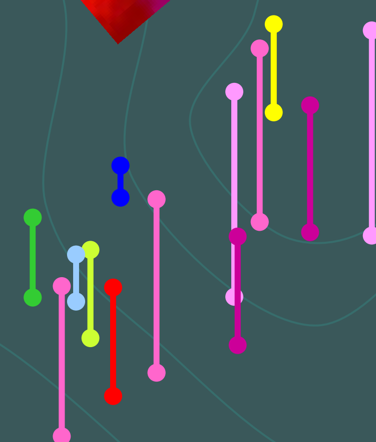
Traditional Differential Geodesy

- Traditionally treated as a problem of fitting data (observations) to a mathematical model of gravity
  - Key model issues: coordinate systems, choice of basis functions, error management, initial conditions
  - Math model is smooth except at density changes
    - Satisfies Laplace equation outside masses: $\nabla^2 V = 0$
    - Satisfies Poisson equation at non-zero densities: $\nabla^2 V = -4\pi\rho G$
- Differential geodesy relates gravity (physics) to curvature (geometry)
- OSU geodesists currently working on problems using traditional differential analysis techniques

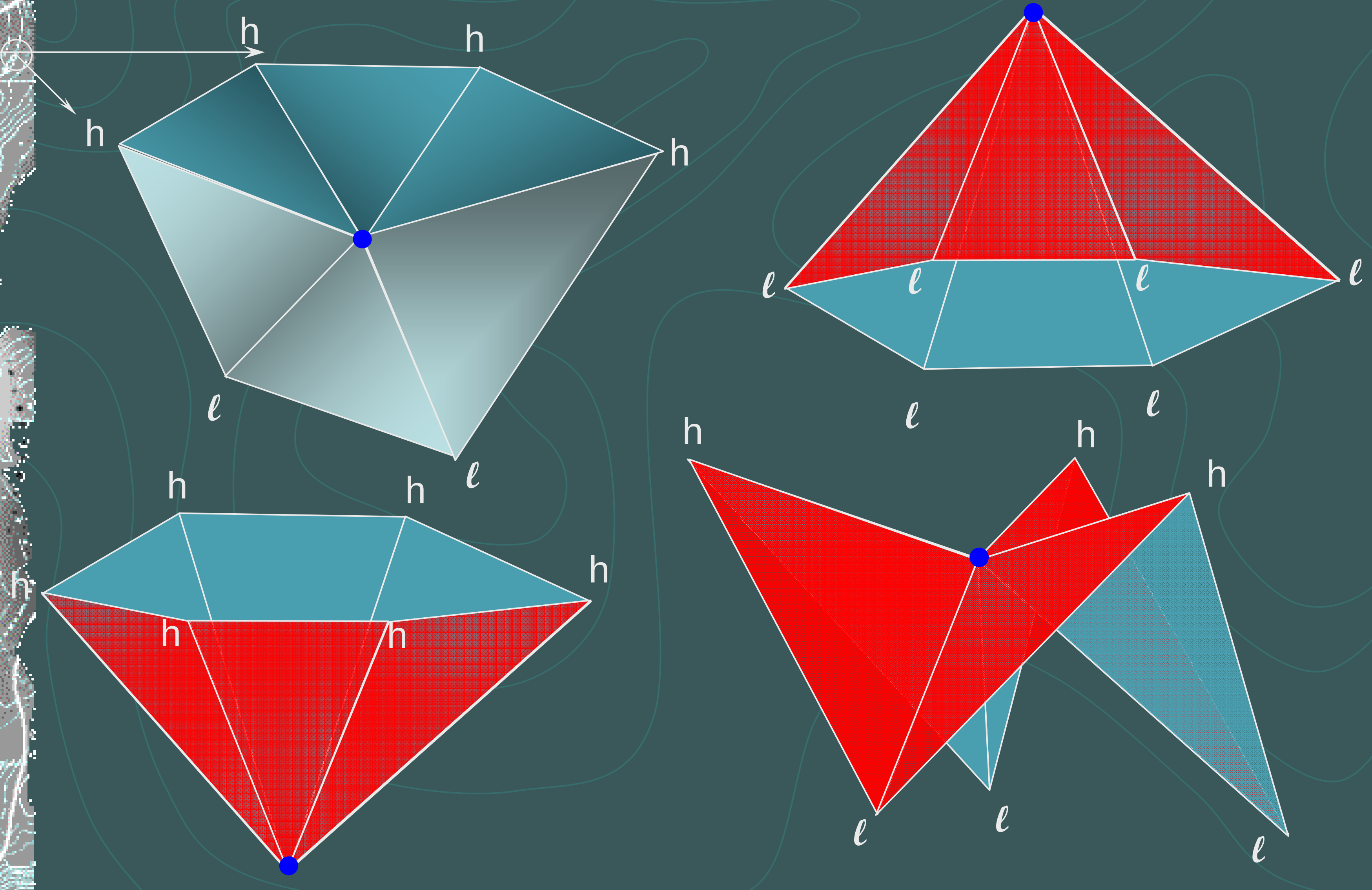
Discrete Differential Geodesy (DDG)

- Discrete Differential Geodesy – what can measurements at sampled points tell us about the structure of equipotential surfaces?
- We will simultaneously apply Discrete Morse Theory to scalar gravity, vector gradients, and tensor (partial derivative) functions



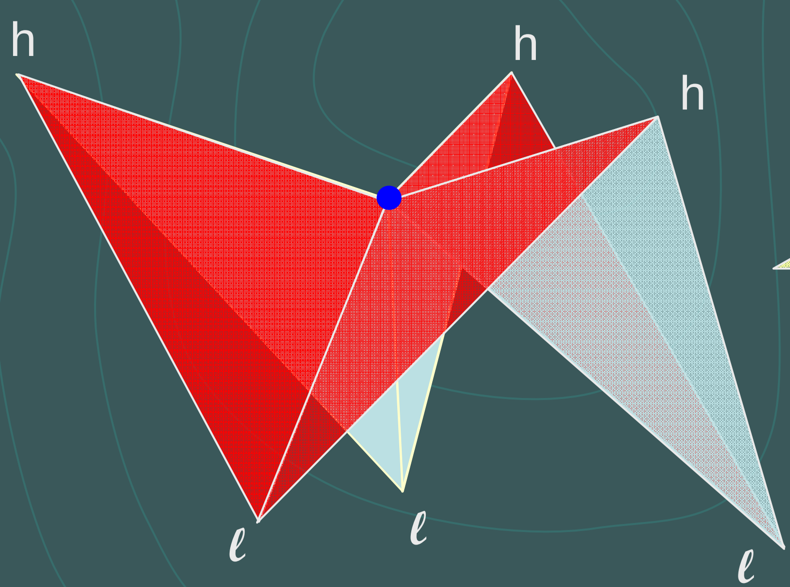


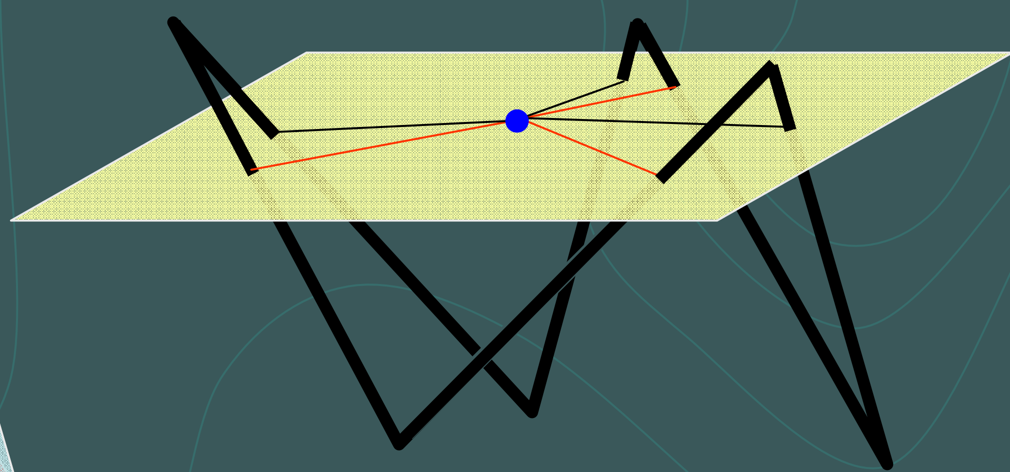
Discrete Morse Theory



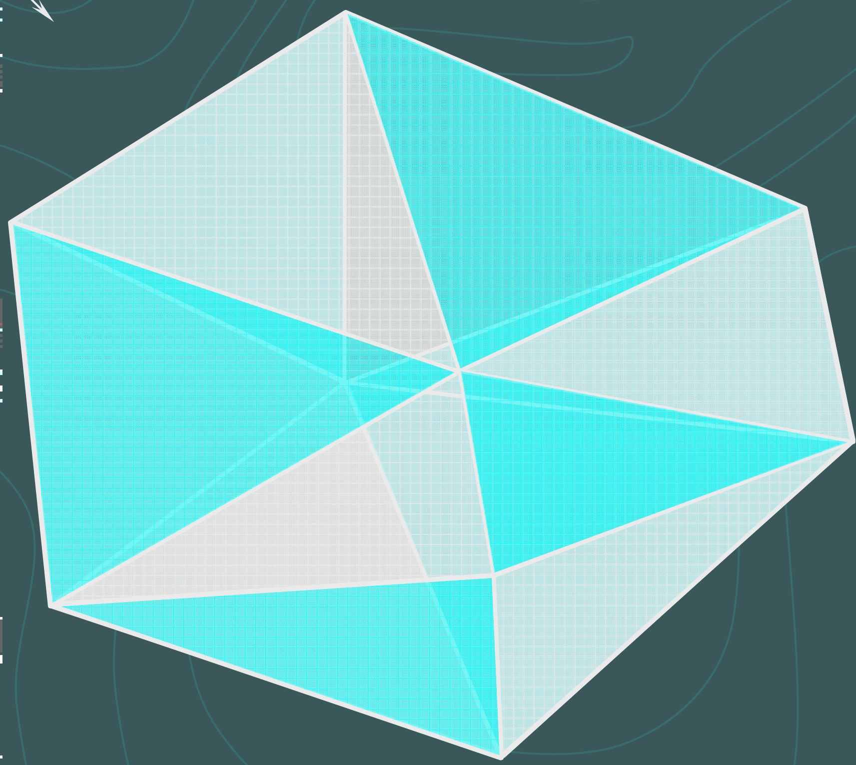
Morse Theory on Polytopes

- Critical points are determined by examining (a) the point's star neighborhood, or (b) the boundary edges of that star neighborhood, or simply (c) the vertex sequence of the neighborhood's boundary cycle

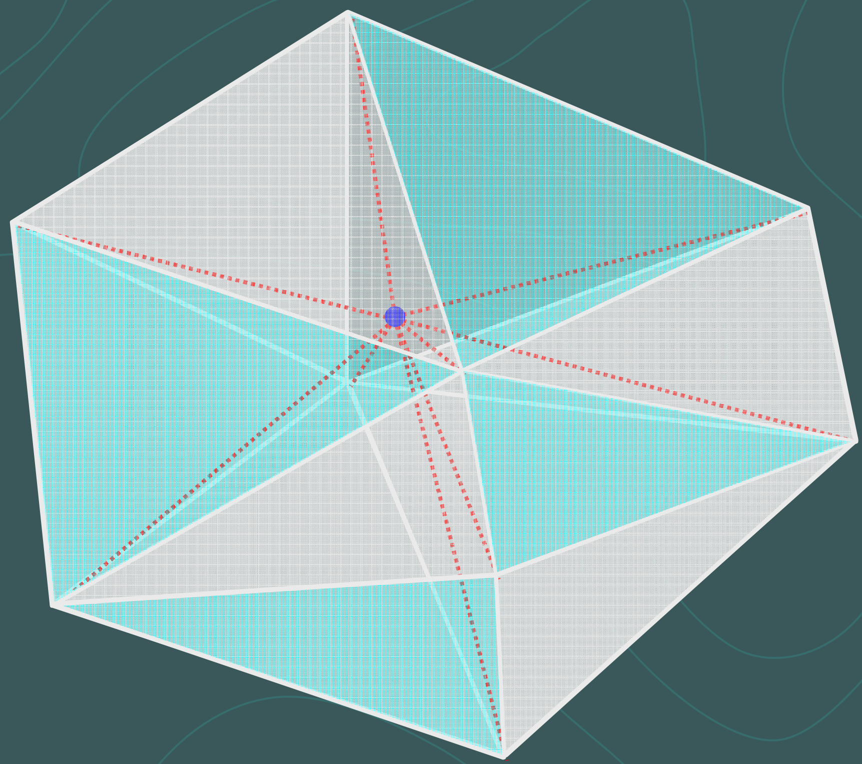




Extending Discrete Morse Theory to sampled functions in 3D



Star polytope with 14 triangular facets, 21 edges, and 9 vertices

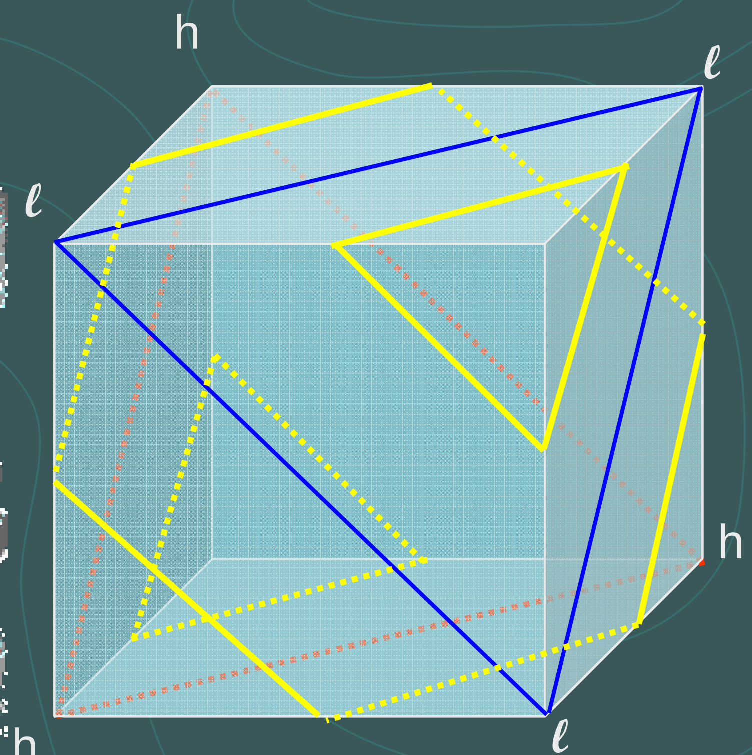


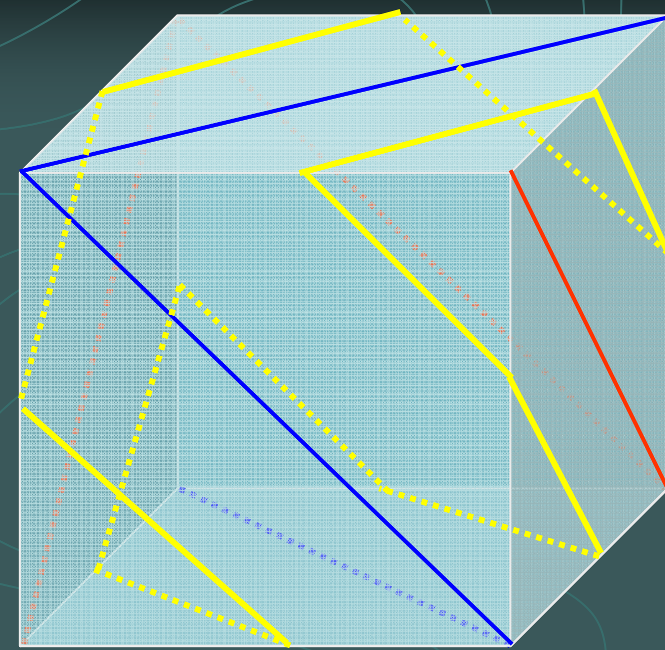
Same polytope as a solid polyhedron with 1 interior vertex, 14 tetrahedra, 21 interior faces, and 9 interior edges

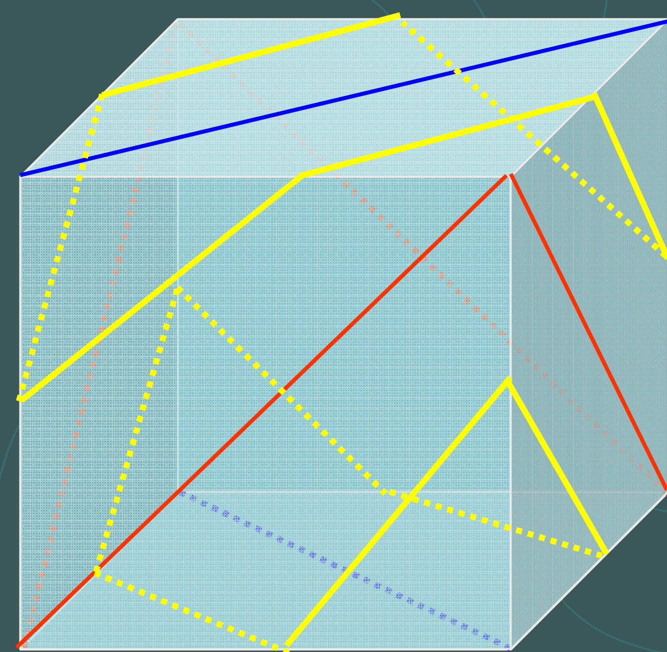
Extending Discrete Morse Theory to sampled scalar-valued functions in 3D

- Work done by Karron (CASI) and Cox (late1990s) on scalar functions defined on sampled volume data: Digital Morse Theory (DMT)
- Local data differences provide estimates for the gradient and other derivatives that may then be used to help disambiguate some not-fully-defined intersection patterns
- When the scalar function is a potential satisfying Laplace's or Poisson's equation, we may also derive and use vector and tensor functions as inputs to our DMT and DDG

Different triangulations yield different iso-curve components for high-low alternating cube vertices

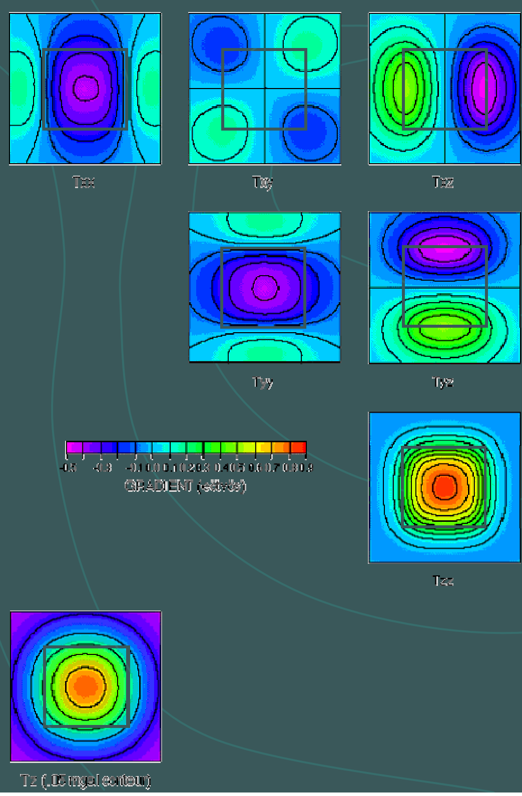


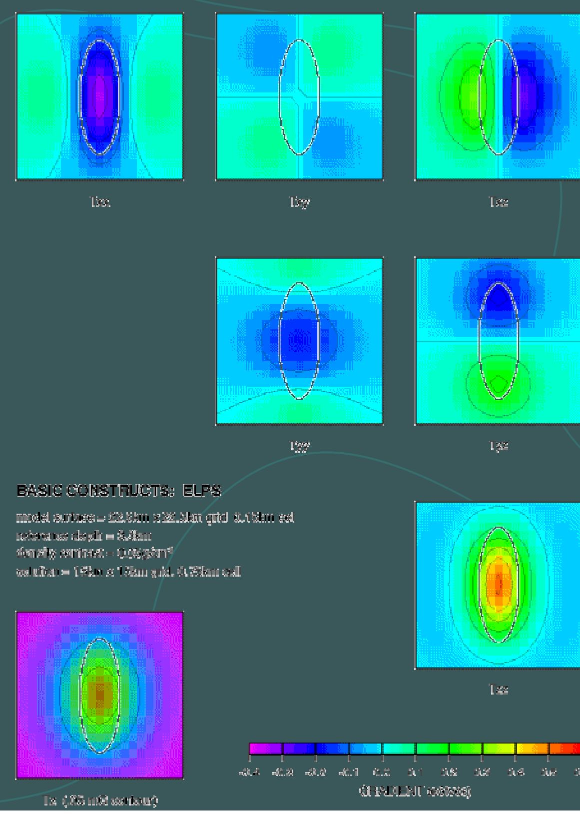




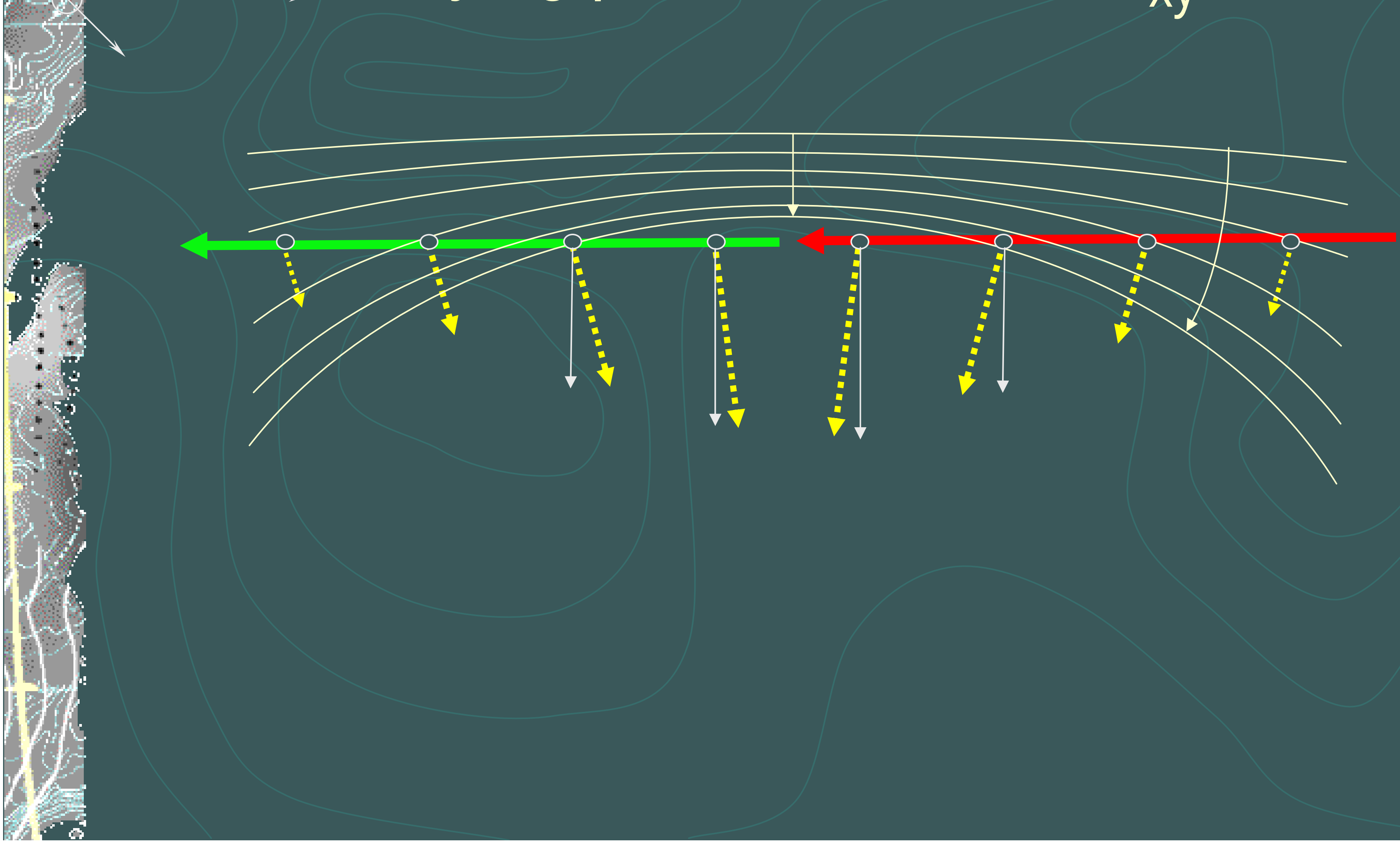
Forward problem (traditional)

- Dissertation research (Lizhi Zhu , PhD student of Chris Jekeli)
  - Study the forward gravity field modeling results under the scope to improve the computation of terrain reductions, downward continuation of gravity anomalies, and orthometric corrections
- Illustrative Models built by Bell Geospace (as shown on their website)



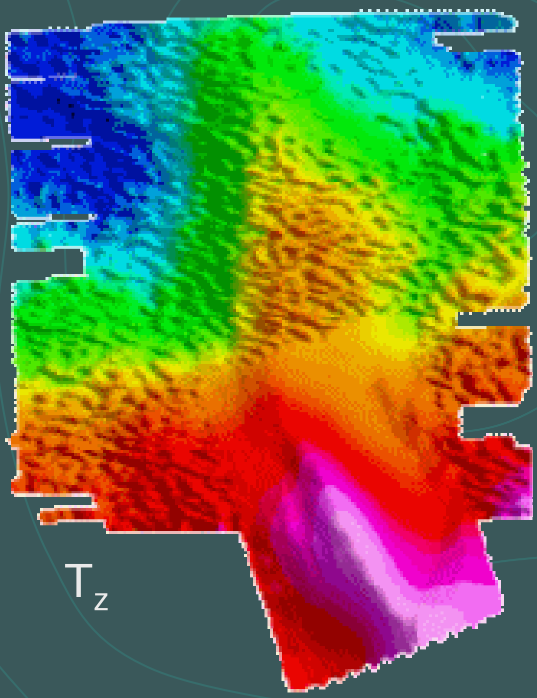


Flying past a mountain: T_{xy}

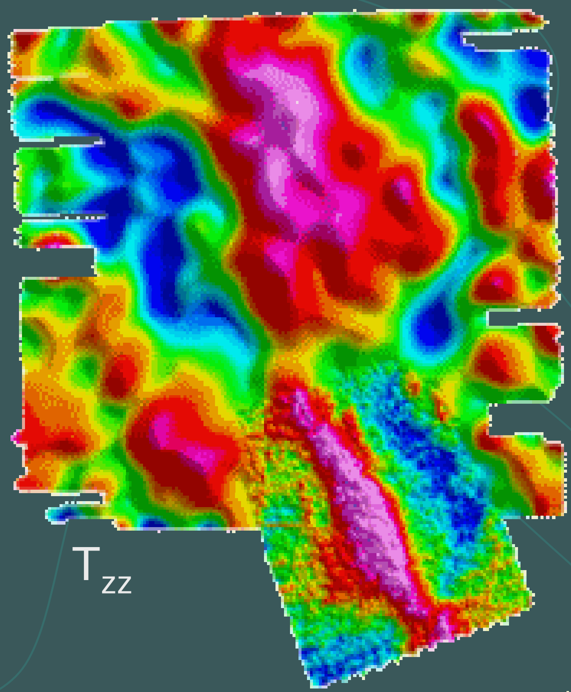


Inverse problem

- Dissertation proposal (Puttipol Dumrongchai under Jekeli):
  - *Small anomalous mass detection from airborne gradiometry*
- Gradient T_z and Tensor component T_{zz} measured by Bell Geospace Gradiometer (as shown on their website)

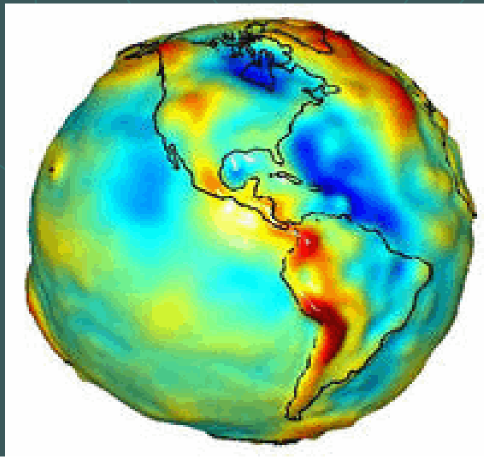


T_z



T_{zz}

- DG in prospecting
  - Following methods described in Nettleton's 1976 book *Gravity and Magnetics in Oil Prospecting*
  - Signatures of salt domes, for example, combine forward and inverse problems
- DG in geodynamics
  - Gravity gradiometry data from San Andreas Fault study
  - Sea surface changes and their gravity field changes
  - GRACE satellite data



Which candidate Morse functions to add to our differential geodesy study?

- Component functions of tensors on a surface or a 3D network of flight lines sampled in $\mathbb{R}^3$
- Gravity gradient tensor $[\partial g_i/\partial x_j]$ (which is equal to):
- Hessian of the potential field $[\partial^2 V/\partial x_i \partial x_j]$
- Tensor components adjusted to remove effects of irregular topographic features
- Functions already studied extensively in the petroleum and mining industries